

Semester Two Examination, 2021

Question/Answer booklet

MATHEMATICS **APPLICATIONS UNITS 3&4**

Section Two: Calculator-assumed

WA student number: Ir

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SOLUTIONS

In words

Your name

Time allowed for this section

Reading time before commencing work: ten minutes Working time:

one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor This Question/Answer booklet Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

drawing instruments, templates, notes on two unfolded sheets of A4 paper, Special items: and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	51	35
Section Two: Calculator-assumed	13	13	100	99	65
				Total	100

Instructions to candidates

- 1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
- 2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
- 3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
- 4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- 5. It is recommended that you do not use pencil, except in diagrams.
- 6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- 7. The Formula sheet is not to be handed in with your Question/Answer booklet.

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Section Two: Calculator-assumed

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9

The table below shows the duration and immediate predecessors for all the activities required to complete a project.

Activity	U	V	W	X	Y	Ζ
Duration (minutes)	17	45	21	11	13	32
Immediate predecessors	—	—	U	V	V, W	Χ, Υ

(a) Construct a project network to show all the above information.

Solution W(21) Y(13) U(17) 45 17 Z(32) 90 0 (0)58 V(45) X(11) 45 **Specific behaviours** ✓ uses dummy edge correctly ✓ adds W and Y with direction and labels to edges

(b) If the meal is to be served at 7.15 pm, determine the latest time the preparations must begin so that the meal is served on time and the path of activities that cannot be delayed.

 Solution

 Minimum completion time is 90 minutes.

 Preparation must begin at 5.45 pm

 Activities on critical path: V, Y, Z.

 Specific behaviours

 ✓ minimum completion time

 ✓ correct start time

 ✓ ordered list of activities on critical path

See next page

(5 marks)

(2 marks)

(2 marks)

(10 marks)

The body length and eye diameter of eight tropical fish of the same species are shown in the table below.

Body length, L cm	6.3	11.9	3.8	8.2	9.1	10.5	5.2	7.5
Eye diameter, D mm	9.3	8.6	9.9	9.1	9.0	8.7	9.8	9.4

(a) Use your calculator to graph the above data and hence describe the direction and strength of the linear association between the variables. (2 mark)

Solution
The association is strong and negative.
Specific behaviours
✓ states strong
✓ states negative

Determine the coefficient of determination for the linear association and interpret its value (b) in context. (2 marks) Solution

Solution
$r^2 = 0.950$
95% of variation in the eye diameter can be explained by the variation in body length.
Specific behaviours
(² to at least 2 desired places

 \checkmark r^2 , to at least 2 decimal places ✓ correct interpretation of coefficient

(c) Determine the equation of the least-squares line to predict D from L and interpret the slope of the line in context.

(3 marks)

	Solution	
D =	-0.17L + 10.55	

The slope means that for every 1 cm increase in length of a fish, the eye diameter is expected to decrease by 0.17 mm.

Specific behaviours

✓ equation, using given variables

- \checkmark indicates as one variable increases, the other decreases
- ✓ quantifies relationship, using units
- (d) Predict the eye diameter of another tropical fish of the same species that has a body length of 5.6 cm and give two reasons that support the validity of this prediction. (3 marks)

Solution
D(5.6) = 9.6 mm
Prediction is valid because correlation is strong, and it does not involve extrapolation.
Specific behaviours
✓ calculates diameter
✓ states strong correlation
\checkmark states interpolation or no extrapolation

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SN108-186-4

A builder bought a scissor lift for \$38 000 and for accounting purposes will depreciate its value by 15% annually.

(a) Show that the value of the lift after one year will be \$32 300.



(b) Write a recursive rule to calculate the value of the lift, T_n in dollars, after *n* years.

Solutio	n
$T_{n+1} = 0.85 \times T_n,$	$T_0 = 38\ 000$
Specific beha	aviours
✓ recursive part	
\checkmark initial term using T_0 =	= 38000 or
$T_1 = 323$	00

(c) Calculate the value of the lift after 4 years.

Solution
$T_4 = \$19\ 836.24$
Specific behaviours
✓ correct value (or rounded to nearest \$)

(d) Deduce the n^{th} term rule for the value of the lift after n years.

Specific behaviours ✓ correct rule

Solution $T_n = 38\ 000(0.85)^n$

(e) Determine the least number of years that must pass for the lift to depreciate by more than 80% of its original value. (2 marks)

Solution
20% of value will remain, so value will be \$7600.
$T_9 = 8801, \qquad T_{10} = 7481.$
Least number of years is 10.
Specific behaviours
✓ indicates required value
✓ correct number of years

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APPLICATIONS UNITS 3&4

(1 mark)

(2 marks)

(1 mark)

(1 mark)

(7 marks)

A reducing balance loan is defined by the recurrence relation $T_{n+1} = a \times T_n - b$, $T_1 = c$, where Tn is the balance of the loan in dollars at the start of month n. The relation was used to create the following spreadsheet.

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Month n	Balance of loan at start of month <i>n</i>	Monthly interest	Monthly repayment	Loan balance carried forward
1	16 500.00	140.25	425.00	16 215.25
2	16 215.25	137.83	425.00	15 928.08
3	15 928.08	135.39	425.00	Х
4		Y		Ζ

(a) Determine the annual percentage interest rate that applies to the loan. (2 marks)

Solution
$140.25 \div 16500 = 0.0085$
$0.0085 \times 12 \times 100 - 10.2\%$ pa
$0.0003 \times 12 \times 100 = 10.270$ pa
Specific behaviours
✓ calculates monthly rate
✓ correct annual rate as percentage

(b) State the value of each of the constants a, b and c in the recurrence relation. (2 marks)

Solution				
a = 1.0085				
$b = 425, \qquad c = 16500$				
,				
Specific behaviours				
\checkmark value of a				
\checkmark value of <i>b</i> and value of <i>c</i>				

(c) Determine the value of *X*, the value of *Y* and the value of *Z* shown in the spreadsheet.

(3 marks)

Solution
$X = 15928.08 + 135.39 - 425.00 = \$15\ 638.47$
$Y = 15638.47 \times 0.0085 = \132.93
X = 15638.47 + 132.93 - 425.00 = \$15346.40
Specific behaviours
✓ value of X
✓ value of Y
\checkmark value of Z

(9 marks)

(2 marks)

A researcher observed a large number of cats, noting for each one its gender and the paw that it used to react to a stimulus placed in front of it. The numbers in each category are shown in the table below.

Cats	Left-pawed	Right-pawed
Male	27	31
Female	28	55

The researcher is interested in whether there is an association between the variables.

(a) Name one of the variables in the study and classify it as numerical or categorical. (2 marks)

Solution
Name: Gender or Paw.
Classifies as Categorical.
Specific behaviours
✓ names one variable
\checkmark classifies variable

(b) Determine the percentage of left-pawed cats that were female.

Solution $\frac{28}{27 + 28} = \frac{28}{55} = 50.9\%$ Specific behaviours ✓ indicates correct total for category ✓ correct percentage (whole number ok)

(c) Use the above data to complete the following table so that it shows the appropriate column or row percentages rounded to the nearest whole number. (3 marks)

Cats (%)	Left-pawed	Right-pawed		
Male	47	53		See table
Female	34	66		Spec ✓ calcula
]	✓ both ro

Solution		
See table		
Specific behaviours		
✓ calculates row %		
✓ both rows add to 100		
✓ correct table rounded		

(d) Explain whether the percentaged table suggests the presence of an association between the variables. (2 marks)

Solution
An association is evident between the variables as the
pairs of percentages in each column (i.e., for each
gender) are quite different.
Specific behaviours
✓ states association
✓ explanation using different percentages

(8 marks)

The table below shows the number of new clients attracted by a business during the first three weeks of an advertising campaign, together with some derived values to assist in the analysis of the time series data.

Week	Day	' (n)	New Clients	Weekly mean	Percentage of weekly mean
	Mon	1	35		A
	Tue	2	40		98.5
1	Wed	3	44	40.6	108.4
	Thu	4	43		105.9
	Fri	5	41		101.0
	Mon	6	42		89.7
	Tue	7	44		94.0
2	Wed	8	52	В	111.1
	Thu	9	49		104.7
	Fri	10	47		100.4
3	Mon	11	49		88.8
	Tue	12	55		99.6
	Wed	13	59	55.2	106.9
	Thu	14	56		101.4
	Fri	15	С		103.3

(a) Calculate the value of *A*, the value of *B* and the value of *C* in the table.

(3 marks)

Solution
$A = 35 \div 40.6 = 86.2\%$
$B = (42 + 44 + 52 + 49 + 47) \div 5 = 46.8$
$C = 57 \times 103.3\% = 57$
Specific behaviours
\checkmark value of A
\checkmark value of B
\checkmark value of C

(b) Determine the seasonal index for Thursdays.

Solution
$I_{THU} = (105.9 + 104.7 + 101.4) \div 3$
$= 312 \div 3$
= 104%
Specific behaviours
✓ uses correct indices
\checkmark calculates index as percent or decimal

(2 marks)

CALCULATOR-ASSUMED

APPLICATIONS UNITS 3&4

(c) The least-squares line to predict the deseasonalised number of new clients c from the day is c = 1.31n + 37.1. Determine the best estimate for the number of new clients expected on Wednesday of Week 4. (3 marks)

Solution
Wednesday of Week 4 is day 18.
c = 1.31(18) + 37.1
= 60.68
$I_{\text{WEDG}} = (108.4 \pm 111.1 \pm 106.9) \pm 3$
-3264 ± 3
$- 520.4 \div 5$ - 100 00/
- 100.8%
$C = 60.68 \times 1.088$
- 66 0
- 00.0
Expect 66 new clients, assuming that the existing trend and
Expect of new clients, assuming that the existing trend and
seasonality continues into the future.
Specific behaviours
✓ uses correct day to calculate deseasonalised number
✓ calculates seasonal index for Wednesday
✓ uses index to calculate whole number estimate

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Question 15

(8 marks)

Ten mains-powered smoke alarms must be installed in a building. The edge weights in the graph below represent the length of cable, in metres, required between adjacent alarms.



(a) Clearly identify the minimum spanning tree on the graph above. (3 marks)

Solution	
See graph	
Specific behaviours	
✓ any tree; ✓6 correct edges; ✓ minimum spanning tree	

(b) Determine the cost of installing the cabling between the alarms using the minimum spanning tree, given that each metre of cabling will cost \$18.50. (2 marks)

SolutionSum of edges:L = 34 + 32 + 15 + 34 + 23 + 21 + 12 + 22 + 29 = 222 mTotal cost: $C = 222 \times 18.50 = 4107 Specific behaviours \checkmark sum of edges \checkmark calculates cost

CALCULATOR-ASSUMED

(c) Explain how your answer to part (b) will change if smoke alarm K is added to the system with cable lengths of 20, 21 and 22 metres to alarms F, G and H respectively. (A copy of the graph from the previous page is shown below if you wish to use it.)
 (3 marks)



Solution	
The minimum spanning tree changes so that edges <i>EF</i> and <i>FG</i> no longer used	
but edges FK, GK and HK are now included.	
The edge sum increases by 6 m to 228 m.	
Total cost increases by \$111 to \$4218.	
Specific behaviours	
✓ states change to edges	
✓ calculates increase in (or new) sum of edges	
✓ calculates increase in (or new) cost	

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SN108-186-4

Question 16

Saltwater flows steadily into a tank, where it is mixed with existing water. An overflow spout on the tank allows excess water to flow out. The salt concentration in the tank can be modelled by $S_{n+1} = 0.96S_n + 850$, $S_0 = 500$, where S_n is the concentration, in parts per million, after saltwater has been flowing into the tank for *n* minutes.

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(a) Determine S_5 .

Solution	
$S_5 = 4331 \text{ ppm}$	
Specific behaviours	
✓ correct value	

Solution n = 15Specific behaviours

(b) Determine the value of n for S_n to first exceed 10 000 ppm.





(d) Describe the feature of the plotted points that indicates the salt concentration will eventually reach a steady-state. (1 mark)

Solution	
Describes 'levelling off' or 'increasing at a decreasing rate', etc.	
Specific behaviours	
✓ reasonable description	

(e) Determine, with justification, the steady-state salt concentration.

Solution		
$S = 0.96S + 850 \Rightarrow S = 21250 \text{ ppm}$		
Specific behaviours		
\checkmark equation, or explanation using term(s) with large n		
✓ correct concentration		

APPLICATIONS UNITS 3&4

(1 mark)

(1 mark)

(2 marks)

CALCULATOR-ASSUMED

Question 17

(7 marks)

Loans are offered by lender A at a rate of 10.32% per annum compounded quarterly and from lender B at a rate of 10.22% per annum compounded daily over the same duration.

(a) Calculate the total amount that must be repaid if \$1500 is borrowed from lender *A* for nine months. (2 marks)

Solution	
10.32	Financial calculator
$A = 1500 \left(1 + \frac{1}{4 \times 100} \right)$	N = 3, I% = 10.32, PV = -1500
A = 1619.12	PMT = 0, P/Y = 4, C/Y = 4
	Solve for $FV = 1619.12$
Hence \$1619.12 must be repaid.	Hence \$1619.12 must be repaid.
Specific behaviours	
✓ shows compound interest formula or values used in financial calculator	
✓ correct amount	

(b) Calculate the total interest that will be charged on a loan of \$90 000 from lender *B* for 30 days. (2 marks)

Solution	
10.22 30	Financial calculator
$A = 90\ 000\left(1 + \frac{365 \times 100}{365 \times 100}\right)$	N = 30, I% = 10.22, PV = -90000
A = 90759.08	PMT = 0, P/Y = 365, C/Y = 365
	Solve for $FV = 90759.08$
Hence \$759.08 interest charged.	Hence \$759.08 interest charged.
Specific behaviours	
✓ shows compound interest formula or values used in financial calculator	
✓ correct amount of interest	

(c) Minimising loan interest is the primary goal for a borrower. Calculate the effective interest rate for each lender and hence recommend which should be chosen. (3 marks)

Lender $A: N = 4$, $APR = 10.32$, $EFF = 10.7263$. Lender $B: N = 365$, $APR = 10.22$, $EFF = 10.7589$. Hence interest will be minimised by choosing lender A	
Lender $B: N = 365$, $APR = 10.22$, $EFF = 10.7589$. Hence interest will be minimised by choosing lender A	
Hence interest will be minimised by choosing lender A	
(as their effective interest rate is less than that of lender <i>B</i>).	
Specific behaviours	
\checkmark effective interest rate for A	
\checkmark effective interest rate for <i>B</i>	
✓ uses effective rates to choose lender	

(7 marks)

When a city bypass is closed, traffic that would normally use it is forced to flow through main roads in the city. The edge weights on the directed graph below show the maximum number of vehicles per minute that can travel between junctions (represented by vertices) without causing congestion in the city.



(a) Determine the value of cut *X* and the value of cut *Y*.

(2 marks)

Solution		
Cut $X = 60 + 60 + 0 + 55 = 175$ vpm. Cut $Y = 65 + 95 = 160$ vpm.		
Specific behaviours		
\checkmark value of cut X		
\checkmark value of cut Y		

(b) Determine the maximum flow of vehicles per hour from *A* to *G*.

(3 marks)

 Solution

 ACFG = 45; ABCFG = 10; ABEFG = 40; ABEG = 10; ADFG = 40; Total = 145.

 OR

 Minimum cut is 55 + 50 + 40 = 145.

Maximum hourly flow is $145 \times 60 = 8700$ vehicles per hour.

Specific behaviours

- ✓ indicates systematic listing
 ✓ correct maximum flow per minute
- ✓ correct maximum flow per minute
 ✓ correct maximum flow per hour
- correct maximum flow per nour
- (c) City engineers recommend taking steps to improve traffic flow between junctions *D* and *E*.
 Determine, with reasoning, the maximum increase in the hourly flow of vehicles from *A* to *G* that their plan could achieve.
 (2 marks)

Solution		
Spare capacity along <i>ADEG</i> is $20 \cdot 0 \cdot 15$, so increase by 15.		
OR		
Minimum cut not using <i>DE</i> is 160, so $160 - 145 = 15$.		
Maximum possible increase is $15 \times 60 = 900$ vehicles/hour.		
Specific behaviours		
✓ reasoning		
✓ correct increase		
See next page		

(8 marks)

A fund with a balance of \$425 000 is used to create an annuity, from which regular withdrawals of \$9500 are to be made at the end of each quarter. Interest at a rate of 6.8% per annum is added to the fund quarterly, just before each withdrawal.

(a) Use one or more calculations to show that the balance of the fund after one withdrawal is \$422 725. (2 marks)

Solution		
$425\ 000 \times 0.068 \div 4 = 7225$	$6.8 \div 4 = 1.7$	
425 000 + 7225 - 9500 = 422 725	425 000 × 1.017 – 9500 = 422 725	
Specific behaviours		
✓ indicates adjustment for interest for one quarter		
✓ calculation: interest less withdrawal or recursive type		

(b) Write a recurrence relation to calculate the balance T_n after the n^{th} withdrawal. (2 marks)

Solution
$$T_{n+1} = 1.017 \times T_n - 9500,$$
 $T_0 = 425000.$ Specific behaviours \checkmark correct recursive part \checkmark correct statement for T_0

(c) Determine the total interest earned by the fund during its first year of operation. (2 marks)

Solution

$$T_4 = 415\ 665.31$$
 $415\ 665.31 + 4 \times 9500 - 425\ 000 = \$28\ 665.31$

 Total interest is \$28\ 665.31.

 Specific behaviours

 \checkmark indicates closing balance

 \checkmark correct total interest

(d) The amount of the regular withdrawal from the fund can be modified so that from the outset, the fund is a perpetuity rather than an annuity. Determine the withdrawal required for this to occur.
 (2 marks)

Solution
Withdrawal is interest for one quarter: $425\ 000 \times 0.068 \div 4 = 7225
Amount of withdrawal is \$7225.
Specific behaviours
✓ indicates withdrawal must be interest for one quarter
✓ correct amount

APPLICATIONS UNITS 3&4

Question 20

(8 marks)

For each quarter in the 12 years from 2006 to 2017, a city library calculated its mean number of users per day and used seasonal indices to deseasonalise the data. A snapshot of the data for the year 2015 is shown in the table below.

Quarter	1	2	3	4
Mean daily users	415.4	465.4	462.6	407.0
Deseasonalised mean daily users	436.5	442.7	431.7	439.8

The trend line for the deseasonalised mean daily users is $\hat{n} = 722 - 8.1t$ where *t* is the quarter and t = 1 corresponds to the first quarter of 2006.

(a) State, with justification, whether the mean number of users per day was increasing or decreasing over the 12 years. (1 mark)



(b) Calculate the seasonal indices and enter them in the following table.

(2 marks)

Quarter	1	2	3	4
Seasonal Index	0.952	1.051	1.072	0.925
Solution				
See table: Decimals to at least 2 dp (or equivalent as percentage).				centage).
$415.4 \div 436.5 = 0.952, 1.051, 1.072, 0.925$				25
Specific behaviours				
✓ at least two correct and indices sum to 4				
✓ all correct				

(c) State, with justification, in which quarter the library was usually busiest during the period that data was collected. (2 marks)

Solution		
Third quarter - the seasonal index is highest for this quarter.		
Specific behaviours		
✓ correct quarter		
✓ justifies using indices		

(d) Determine an estimate, to the nearest whole number, for the mean daily users the library experienced in the first quarter of 2017. (3 marks)

Solution
t = 45
$\hat{n} = 722 - 8.1(45) = 357.5$
$357.5 \times 0.952 \approx 340$
Estimate there were 340 users.
Specific behaviours
\checkmark correct value of t
✓ calculates \hat{n}
✓ seasonalises to obtain correct estimate

APPLICATIONS UNITS 3&4

Question 21

(7 marks)

An electronics store advertises a TV for sale at a price of \$5950. Rather than pay this amount in full, the store offers customers a no deposit reducible balance loan with 24 monthly payments of \$281.20. The first repayment is due one month after the customer makes the purchase, just after interest for the month is added to the loan balance.

(a) Determine the total interest paid by customers who buy the TV using the loan. (2 marks)

 Solution

 Repay: 24 × 281.20 = \$6748.80

 Interest: 6748.80 - 5950 = \$798.80

 Specific behaviours

 ✓ calculates total repaid

 ✓ calculates interest

(b) Determine the annual percentage interest rate that applies to the loan.

(2 marks)

Solution				
Using F	inancial ap	p:		
	<i>N</i> = 24,	PV = 5950,	PMT = -281.20,	
	FV = 0,	P/Y = 12,	C/Y = 12	
Solve fo	or $I\% = 12.4$	4, so rate is 12.	4% per annum.	
Specific behaviours				
✓ shows correct values for financial calculator				
✓ corre	ct annual ra	ate		

A customer decides to buy the TV using a similar reducible balance loan to that offered by the store but financed by their bank at an annual interest rate of 13.98% and over 36 months.

(c) Determine their monthly repayment.

(2 marks)

		Solution	
Using Financial a	pp with:		
Ν	= 36,	PV = 5950,	<i>I</i> % = 13.98,
FV	/ = 0,	P/Y = 12,	C/Y = 12
Solve for <i>PMT</i> =	-203.30,	so their repay	vment is \$203.30 per month.
Specific behaviours			
✓ shows correct values for financial calculator			
✓ correct repaym	nent		

(d) Determine the total interest that the bank will charge on the loan over the 36 months.

(1 mark)

Solution
Using Financial app: $\Sigma INT = -1368.77$.
Using $36 \times 203.30 - 5950 = 1368.80$.
Total of \$1368.77 or \$1368.80 in interest.
Specific behaviours
✓ correct amount

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

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